MATH1009 – Final (P2) – Retake
Wednesday, June 05, 2019
Instructor: Abuzer Yakaryılmaz

Name and surname:

Student number:

Questions

(120 minutes / 10 points in total)

1. (2 points) For the matrix $C = \begin{bmatrix} 2 & -2 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, find $\det C$, $\text{adj}\, C$, and then C^{-1} by using the formula $C^{-1} = \frac{1}{\det C}\text{adj}\, C$.

2. (2 points) Let $z = 2 + i$. Express the complex number $\dfrac{1}{z} + \dfrac{1}{\overline{z}}$ in the form of $x + iy$.

3. (1 point) Express the complex number $e^{i(-\frac{\pi}{3})}$ in the form of $x + iy$.
   Remember that $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ and $\cos\frac{\pi}{3} = \frac{1}{2}$.

4. (2 points) Completely factor the polynomial $x^3 + 2x^2 - x - 2$.

5. (2 points) Let $\star$ be a binary operation on the set $S = \{0, 1, 2, 3, 4, 5, 6\}$ defined as

$$a \star b = a + b \mod 7,$$

   where $a, b \in S$.
   a) Calculate $3 \star 3$, $4 \star 4$, $5 \star 5$, and $6 \star 6$.
   b) Find 3^{-1} and 5^{-1}.

6. (1 point) Let $*$ be a binary operation on the positive real numbers defined as

$$a * b = \frac{ab}{3},$$

   where $a, b \in \mathbb{R}^+$. What is the identity element for $*$?

During the exam, please do not use any electronic device or any course (related) material.