MATH1009 – Midterm (P1)
Wednesday, March 06, 2019
Instructor: Abuzer Yakaryılmaz

Name and surname:

Student number:

Questions

(90 minutes / 10 points in total)

1. (1 point) Calculate $\begin{vmatrix} 2 & -2 \\ 1 & 6 \end{vmatrix}$.

2. Let $A = \begin{bmatrix} 3 & a \\ -2 & -4 \end{bmatrix}$ be a matrix.
   a) (1 point) If A is singular, then what is the value of a?
   b) (1 point) Calculate A^{-1} for $a = 9$.

3. (2 points) If $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = -2$, then calculate the following determinant:

$$\begin{vmatrix} 0 & 0 & 3 & 0 \\ 0 & a & 3 & b \\ 2 & -2 & -6 & 4 \\ 0 & c & -5 & d \end{vmatrix}.$$

4. Let $B = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$ be an integer matrix.
   (a) (2 points) Find B^{-1}.
   (b) (1 points) Solve the following system of linear equations:

$$\begin{array}{rcrcrcrcr} x & + & y & + & z & + & t & = & 2 \\ x & - & y & + & z & + & t & = & 4 \\ x & + & y & - & z & - & t & = & 6 \\ x & - & y & - & z & + & t & = & 10 \end{array}$$

   (Hint: You may use the answer in (a) to solve the system.)

5. (2 point) For the matrix $C = \begin{bmatrix} 1 & -1 & 2 \\ 2 & -2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$, find $\det C$, $\operatorname{adj} C$, and then C^{-1} by using the formula $C^{-1} = \frac{1}{\det C} \operatorname{adj} C$.

During the exam, please do not use any electronic device or any course (related) material.