

MATH1009 – Course Exam  
Friday, February 22, 2019  
Instructor: Abuzer Yakaryılmaz

Name and surname:

Student number:

**Questions**

(90 minutes / 70 points in total)

1. (10 points) If  $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 3$ , then calculate the following determinant:

$$\begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & -2 & 3 & 4 \\ 0 & -6 & 2a & 2b \\ 0 & -5 & 3c & 3d \end{vmatrix}.$$

2. Let  $A = \begin{pmatrix} 2 & 2 & 2 & 2 \\ 2 & -2 & 2 & 2 \\ 2 & 2 & -2 & -2 \\ 2 & -2 & -2 & 2 \end{pmatrix}$  be an integer matrix.

(a) (10 points) Find  $A^{-1}$ .

(b) (10 points) Solve the following system of linear equations:

$$\begin{aligned} x + y + z + t &= 2 \\ x - y + z + t &= 4 \\ x + y - z - t &= 6 \\ x - y - z + t &= 10 \end{aligned}$$

(Hint: You may use the answer in (a) to solve the system.)

3. (10 points) Find all six roots of  $z^6 = 1$ . Each root should be represented in the form either  $x$  or  $x + iy$ , where  $x$  and  $y$  are some real numbers. Note that  $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$  and  $\cos \frac{\pi}{3} = \frac{1}{2}$ .
4. (10 points) Determine the unique polynomial of degree  $< 3$  passing through (satisfying) the three points  $(0, -1)$ ,  $(1, -2)$ ,  $(3, 10)$ .
5. (10 points)  $p(x)$  is a real polynomial with degree 5. It has one real and four complex roots. If its three roots are

$$x_1 = 1, x_2 = 1 + 2i, x_3 = 3 - i,$$

find  $p(x)$  (write  $p(x)$  in the form  $ax^5 + bx^4 + cx^3 + dx^2 + ex + 1$ ).

[Hint: If  $f(z)$  is a polynomial with real coefficient, then its non-real roots occur in complex-conjugate pairs, i.e. if  $f(z) = 0$ , then  $f(\bar{z}) = 0$ .]

6. (10 points) Let  $*$  be the operation on the set  $\mathbb{R}$  of real numbers defined by

$$a * b = a + b + 2ab.$$

- (a) Find  $2 * 3$ ,  $3 * (-5)$ , and  $7 * (\frac{1}{2})$ .
- (b) Is  $(\mathbb{R}, *)$  a semigroup? Is it commutative?
- (c) Find the identity element of  $*$ .
- (d) Which elements have inverses and what are they?

**During the exam, please do not use any electronic device or any course (related) material.**