MATH1009 – Complex numbers
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Problem set - 1

Let $z_1 = 2 - i$ and $z_2 = 1 + 3i$ be two complex numbers.

1. What are $\operatorname{Re} z_1$, $\operatorname{Im} z_1$, $\operatorname{Re} z_2$, and $\operatorname{Im} z_2$?
2. Calculate $z_1 + z_2$, $z_1 - z_2$, $-2z_1$, and $2z_1 - 3z_2$.
3. Find $\overline{z_1}$ and $\overline{z_2}$ (conjugate of each complex numbers).
4. Calculate $z_1\overline{z_1}$ and $\overline{z_2}z_2$.
5. Calculate $z_1 z_2$, $z_1\overline{z_2}$, and $\overline{z_1}z_2$.
6. If $a + bi = \dfrac{1}{z_1}$, then find a and b.
7. If $a + bi = \dfrac{1}{z_2}$, then find a and b.
8. If $a + bi = \dfrac{1}{z_1} + \dfrac{1}{z_2}$, then find a and b.
9. If $a + bi = \dfrac{1}{z_1} - \dfrac{1}{z_2}$, then find a and b.
10. If $a + bi = \dfrac{2}{1+i} - \dfrac{1}{3-i} + \dfrac{3}{2+i}$, then find a and b.
11. If $a + bi = \dfrac{1-2i}{3-4i}$, then find a and b.
12. If $a + bi = \dfrac{z_1}{z_2}$, then find a and b.
13. Calculate z_1^2, z_1^3, and z_1^4.
14. Calculate $z_1^2 z_2^3$.
15. If $a + bi = \dfrac{1}{z_1^2} + \dfrac{1}{z_2^2}$, then find a and b.
16. Find the complex-valued roots of the equation $x^2 + 4x + 5 = 0$.
17. Find the complex-valued roots of the equation $2x^2 + 2x + 1 = 0$.

Problem set - 2

- $\sin 0 = 0$ and $\cos 0 = 1$.
- $\sin\frac{\pi}{6} = \frac{1}{2}$ and $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$.
- $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ and $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$.
- $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ and $\cos\frac{\pi}{3} = \frac{1}{2}$.
- $\sin\frac{\pi}{2} = 1$ and $\cos\frac{\pi}{2} = 0$.

1. Find $|1+i|$, $|-2-3i|$, $|4i|$, $|5|$, and $|4i-3|$.
2. Find $|z|$ when $z = \dfrac{(1-i)^4(-3-4i)^2}{(\sqrt{3}-i)^3(2-\sqrt{21}i)}$.
3. Find $\mathrm{Arg}(1)$, $\mathrm{Arg}(i)$, $\mathrm{Arg}(-1)$, and $\mathrm{Arg}(-i)$.
4. Find $\mathrm{Arg}(1+i)$, $\mathrm{Arg}(-1+i)$, $\mathrm{Arg}(-1-i)$, and $\mathrm{Arg}(1-i)$.
5. Find $\mathrm{Arg}(\sqrt{3}+i)$ and $\mathrm{Arg}(1+\sqrt{3}i)$.
6. Find $\mathrm{Arg}(-3\sqrt{3}-3i)$ and $\mathrm{Arg}(-2+2\sqrt{3}i)$.
7. Express the following complex numbers in modulus-argument form:
$$(1+i),\ (-1+i),\ (-1-i),\ \text{and } (1-i).$$
8. Express the following complex numbers in modulus-argument form:
$$(\sqrt{3}+i),\ (1+\sqrt{3}i),\ (-3\sqrt{3}-3i),\ \text{and } (-2+2\sqrt{3}i).$$
9. Express the these complex numbers $(1+i)^2$, $(1+i)^4$ $(1+i)^6$, and $(1+i)^8$ in the form of $x+iy$ by using modulus-argument form.
10. Express the complex number $(-2+2\sqrt{3}i)$ in the form of $x+iy$ by using modulus-argument form.
11. Express the complex numbers $e^{i2\pi}$, $e^{i\pi}$, $e^{i\frac{\pi}{2}}$, and $e^{i\frac{\pi}{2}}$ in the modulus-argument form.
12. Express the complex numbers $e^{i\frac{\pi}{3}}$, $e^{i\frac{2\pi}{3}}$, $e^{i\frac{4\pi}{3}}$, and $e^{i\frac{5\pi}{3}}$ in the modulus-argument form.
13. Express all roots of $z^3 = 1$, $z^4 = 1$, $z^6 = 1$, and $z^{12} = 1$ in the modules-argument form and then in the form of $x+iy$.

    Remark that $e^0 = e^{i2\pi} = e^{i4\pi} = e^{i6\pi} = e^{i8\pi} = \cdots = e^{i2k\pi} = 1$ for any $k \in \mathbb{Z}$.