EXAM 2 (duration: 40 minutes)

DatZ1143: Discrete mathematics for computing
Instructor: Abuzer Yakaryılmaz
Autumn 2019

Name and surname:
Date: October 03, 2019

Questions

(There are 4 questions with total 10 points.)

1. Let $P(x)$ denote the statement "$x^2 \leq x + 1$", where x is an integer.
   1.a. Give an example of x such that $P(x)$ is true (1 point)

   1.b. Give an example of x such that $P(x)$ is false (1 point)

2. Determine the truth value of each of these statements if the domain of each variable consists of all integers. Write your answer after each statement. If the statement is false, then give a counter-example that makes the statement false. (1 point each, 4 points in total)

- $\forall y \forall z((y + z + 1)^2 > 0)$

- $\forall x \exists y(x^2 = y)$

- $\forall x \exists y(x = y^2)$

- $\forall y \exists z(y + z - 2 = 0)$

3. Write the negation of the statement

$$\forall x \exists y \, (x + y < 0) \, ,$$

where both x and y are positive integers. (2 points)

4. (corrected) Show that the compound proposition

$$(p \to (p \to \neg p)) \wedge ((r \leftrightarrow r) \to p)$$

is a contradiction. (2 points)