Name and surname: **Date: January 14, 2020**

RETAKE EXAMS

DatZ1143: Discrete mathematics for computing
Instructor: Abuzer Yakaryılmaz
Autumn 2019

EXAM 1

(There are 3 questions with total 10 points.)

1) Construct a truth table for the compound proposition $(p \to q) \wedge (\neg p \to \neg q)$. (5 points)

2) Show that the statement $(q \wedge \neg q) \to (r \to r)$ is a tautology. (3 points)

3) Show that the statement $(q \vee p \vee r \vee \neg q) \to (q \wedge p \wedge r \wedge \neg q)$ is a contradiction. (2 points)

EXAM 2

(There are 3 questions with total 10 points.)

1) Let $P(x)$ denote the statement "$x^2 + 1 \leq 2x$", where x is an integer.
1.a) Give an example of x such that $P(x)$ is true (2 points)
1.b) Give an example of x such that $P(x)$ is false (2 points)

2) Give a counter example to show that the statement $\forall y \left((2y+4)^2 - 2 > 0\right)$ is false, where the domain of y is the set of integers. (3 points)

3) Name at most three statements from the following list that are true, where the domain of each variable is the set of integers. For each false statement you will name, you will get -1 point. (3 points)

3.a) $\forall x \exists y (x^2 = y)$

3.b) $\forall y \exists x (x^2 = y)$

3.c) $\forall x \forall y (2x^2 + 3y^2 > 0)$

3.d) $\forall x \exists y (x - y - 1 = 0)$

3.e) $\forall x (-x \neq x^3)$

3.f) $\exists x \exists y \exists z (x^2 + y^2 = z^2)$ where x, y, z are non-zero integers

EXAM 3

(There are 3 questions with total 10 points.)

1) Write down all subsets of the set $S = \{a, b, c, \{a, b, c\}\}$. (3 points)

2) The functions $f(a) = \left\lceil \frac{a}{4} \right\rceil$ and $g(a) = \left\lfloor \frac{a}{4} \right\rfloor$ are defined from integers to integers. Calculate the values of the following two summations.

2.a) $f(-6) + f(-3) + f(0) + f(3) + f(6) =?$ (2 points)

2.b) $g(-6) + g(-3) + g(0) + g(3) + g(6) =?$ (2 points)

3) Let $f(x) = 2x + 5$ and $g(x) = 3x - 7$ be two functions from integers to integers. Calculate the values of $(f \circ g)(2)$ and $(g \circ f)(-2)$. (3 points)

EXAM 4

(There are 3 questions with total 10 points.)

1) Calculate the summation $\sum_{j=1}^{50} 4j - 3$. Your answer must be a single integer. (3 points)

2) Calculate the following **infinite** summation:

$$4^3 + 4^2 + 4^1 + 4^0 + \frac{1}{4^1} + \frac{1}{4^2} + \frac{1}{4^3} + \cdots + \frac{1}{4^n} + \cdots \ .$$

Your answer must be either a single number or the summation of two numbers. (3 points)

3) Which pairs in the following list

$$\{-80, -70, -60, -50, -40, -30, -20, -10, 10, 20, 30, 40, 50, 60, 70, 80\}$$

are congruent to each other modulo 9? (4 points)

EXAM 5

(There are 3 questions with total 10 points.)

1) Use mathematical induction to prove that the following summation

$$2 \cdot 1 + 2 \cdot 3 + 2 \cdot 3^2 + 2 \cdot 3^3 + \cdots + 2 \cdot 3^n$$

is equal to $3^{n+1} - 1$. (4 points)

2) Use mathematical induction to prove that the following summation

$$1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + 3 \cdot 4 \cdot 5 + \cdots + n(n+1)(n+2)$$

is equal to $n(n+1)(n+2)(n+3)/4$. (3 points)

3) Use mathematical induction to prove that $n^2 - 8n - 1 > 0$ for every integer $n \geq 9$. (3 points)

EXAM 6

(There are 3 questions with total 10 points.)

1) How many bit strings are there of length eight of less? (2 points)

2) How many positive integers less than 1000 are divisible by 9 but not by 7? (3 points)

3) A bowl contains 13 pink balls and 11 orange balls. You select balls at random without looking at them.
3.a) How many balls must you select to be sure of having at least five balls of the same color? (3 points)
3.b) How many balls must you select to be sure of having at least six orange balls? (2 points)

EXAM 7

(There are 3 questions with total 10 points.)

1) How many permutations of the letters ABCDEFGH contain the string AH and end with H? (2 points)

2) Remark that bit strings are different than binary numbers.
2.a) How many **bit strings** of length six have exactly three 1s? (2 point)
2.b) How many six-digit **binary numbers** have exactly three 1s? (2 points)

3) How many strings of six decimal digits have exactly three digits that are 5s? (3 points)

EXAM 8

(There are 3 questions with total 10 points.)

1) What is the probability that a positive integer not exceeding 100 selected at random is divisible by 3 or 5? (3 points)

2) We have a biased coin such that the probability of getting head is four times of the probability of getting tail. What is the expected number of getting tail if we toss this biased coin ten times? (3 points)

3) Find the probability that a randomly generated bit string of length 8 contains exactly five 0s and three 1s. (4 points)