

**MATH1009 – Final Exam 2**  
**Wednesday, December 21, 2016**  
**Instructor: Abuzer Yakaryilmaz**

Name and surname:

Student number:

**Questions**  
(120 minutes)

1. (10 points)  $\frac{2+i}{1-2i} = x + iy$  for some real numbers  $x$  and  $y$ . Find  $x$  and  $y$ .
2. (15 points) Find all three roots of  $z^3 = 1$ . Each root should be represented in the form either  $x$  or  $x + iy$ , where  $x$  and  $y$  are some real numbers. Note that  $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$  and  $\cos \frac{\pi}{3} = \frac{1}{2}$ .
3. (20 points) Determine the unique polynomial of degree  $< 3$  passing through (satisfying) the three points  $(-1, 10)$ ,  $(0, 5)$ ,  $(1, 6)$ .
4. (15 points)  $p(x)$  is a real polynomial with degree 4. It has four complex roots. If its two roots are

$$x_1 = -i \text{ and } x_2 = 2i,$$

find  $p(x)$  (write  $p(x)$  in the form  $ax^4 + bx^3 + cx^2 + dx + 1$ ).

5. (15 points) If  $x^2 - 1$  is a factor of  $x^3 - 3x^2 + ax + b$ , then find  $a$  and  $b$ .
6. (25 points) Let  $*$  be the operation on the set of integers  $\mathbb{Z}$  defined by

$$a * b = a + b - 2ab.$$

- (a) (4 points) Find  $1 * 2$  and  $2 * (-1)$ .
- (b) (5 points) Is  $(\mathbb{Z}, *)$  a semigroup?
- (c) (3 points) Is it commutative?
- (d) (5 points) Find the identity element of  $*$ .
- (e) (8 points) Find  $(-2)^{-1}$ ,  $(-1)^{-1}$ ,  $1^{-1}$ , and  $2^{-1}$  if exist.